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SOME ASPECTS OF TURBULENT SCATTERING OF ELECTROMAGNETIC WAVES BY HYPERSONIC WAKE FLOWS*

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ABSTRACT

The present analytical study of the turbulent scattering of electromagnetic waves is directed to certain features of the phenomenon considered as peculiar to hypersonic wakes. Effects of flow intermittency, non-isotropy of the turbulence structure, and finite width of the wake on the scattering cross section in its frequency and aspect-angle dependence are of primary concern.

Phenomena of turbulence revelant to the scattering problem are briefly discussed. In particular, it is shown that electron-density fluctuations of a turbulent nature, in addition to those caused by "turbulence", will be produced by the intermittency behavior of turbulent wake flows. This fluctuation is found to depend on the mean electron-density distribution, and Townsend's intermittency "δ" function.

Phenomenological consideration of the physical process by which the turbulent electron-density fluctuations are likely to be produced in hypersonic wakes yields an expression for the intensity of the fluctuations in terms of the gradient of the mean electron-density distribution, the intensity of the turbulent velocity fluctuation, and a turbulence scale of the turbulent velocity field.

Application of the above considerations to turbulent scattering by underdense wakes has been made. In contrast to the ionospheric scattering, two contributions to the scattering cross section are obtained in the present case: the first one arises from the intermittency phenomenon and vanishes if the intermittency is omitted, and the second one is due to "turbulent" fluctuations. The frequency-dependence of these two contributions are seen to be quite different. It is further shown that flow intermittency, non-isotropy of turbulence structure, and finite width of the wake will introduce aspect-angle dependence into the scattering cross section. A possible form of the total scattering cross section for hypersonic wakes is also given.

Much of the problem remains to be studied. Areas of additional work of immediate concern are briefly described.

I. INTRODUCTION

Interest in turbulence as a mechanism for scattering of electromagnetic waves probably began when Booker and Gorden[1] suggested it to explain some anomalous phenomena of radio scattering in the troposphere. The original theory developed by Booker and Gorden formed the basis for further theoretical and experimental work[2,3,4] in many related scattering problems. As applied to ionosphere back-scattering[3], the essence of this theory is: when the scattering volume is much larger than λ_o^3, where λ_o is the wave length of the incoming electromagnetic wave, and when the electron density is such that the plasma frequency is less than the wave frequency, (i.e. the plasma is underdense), turbulent irregularities in electron density responsible for the back-scattering are those with wave vector equal to $2\bar{k}$, where $\bar{k}$ is the wave vector of the incident electromagnetic wave.

In determining the scattering cross-section, however, assumptions concerning the intensity of the electron density fluctuations and the form of the correlation or spectrum function must be made on account of insufficient knowledge about the turbulence. Several different formulas have been proposed for the scattering cross section by different workers, but none of these appear to be entirely satisfactory.[5] Hence, at the present time the theory of turbulent scattering of radio waves in the ionosphere must be considered as still incomplete.

It is also recognized that turbulent scattering may be responsible for radar return signals from the wakes of high-speed reentry objects at certain altitudes. Experimentally, the back-scattering signals from the wake of a reentry object have been found to be undetectable by radar until the object is at an altitude at which the wake apparently becomes turbulent. It is reasonable to expect no significant back-scattering returns from a laminar wake at small aspect angles with respect to the propagation direction of the radar wave (Figure 1), (this problem was investigated in reference 6 based on a simplified model).

The problem of turbulent scattering by underdense wakes has been studied in references 7, 8, 9 and others. The theoretical basis of these studies can be traced back to the work of Booker and Gorden[1], and does not appear to have progressed much beyond Booker's recent work[3] on ionspheric radio propagation. The purpose of this study is to examine the physical nature of the scattering problem peculiar to the wakes of reentry objects of certain type. A new, but tentative, form of the scattering cross section has been obtained. Since solution to this problem is not yet complete, this report may serve to define the scope and present status of this problem, and also areas of future work.

II. SCATTERING OF ELECTROMAGNETIC WAVES BY TURBULENT MEDIUM

In the theory originated by Booker and Gorden[1], irregular fluctuations of the dielectric constant of a medium are considred as the sources for scattering of electromagnetic waves by the medium. As in the turbulence theory[10], the dielectric constant of a medium may be separated into its mean value K and its flucation ΔK, where both K and ΔK may be functions of the space coordinates $\bar{r}$ (or x_i, $i = 1, 2, 3$) and the time t.

When a plane trasverse electromagnetic wave of the form $\bar{A}_o \exp\{-i(\omega t - \bar{k}\cdot\bar{r})\}$ is incident on a volume V of such turbulent medium, the scattering cross section (per unit solid angle) can be written in the following form

$$\sigma = \frac{k^4 \sin^2 \beta}{16\pi^2} \int_V \int_V \left\langle \frac{\Delta K}{K}(\bar{r}_1) \frac{\Delta K}{K}(\bar{r}_2)\, e^{i(\bar{k} - k\bar{m})\cdot(\bar{r}_1 - \bar{r}_2)} \right\rangle dV_1\, dV_2\,, \tag{1}$$

where $\bar{m} = (\bar{r}/r)$ is a unit vector directed from the origin of coordinates (chosen within the scattering volume V) to the observation point (Figure 2), β is the angle between $\bar{A}_o$ and $\bar{m}$, and the mean value of a quantity is indicated by the symbol < > which yields, in the present case, the

correlation function for the dielectric-constant fluctuations. The above formula can be easily deduced from Tatarski's results.[4]

In obtaining Eq. (1), a perturbation technique, in principle the same as "Born approximation" in its first approximation, has been used to solve the Maxwell equations. As a consequence, Eq. (1) becomes invalid if K vanishes in the scattering volume. Let the dielectric constant be of the form

$$\frac{K}{K_v} = 1 - \frac{\omega_p^2}{\omega^2} , \tag{2}$$

where K_v denotes the value of K in vacuum, and ω_p is the plasma frequency. Thus, Eq. (1) applies only to "underdense" plasmas, i.e. $\omega_p << \omega$. In addition, since both $\bar{A}_o$ and k are taken as constant in the scattering volume, the quantity grad $(\ln K)$ must remain small. However, the fluctuation in electron density Δn is allowed to be comparable to N, the mean electron density. This is due to the fact that

$$\frac{\Delta K}{K_v} = \frac{\omega_p^2}{\omega^2} \frac{\Delta n}{N} \tag{3}$$

and will remain small compared to 1 for $\Delta n \sim N$ as long as $\omega_p^2 << \omega^2$. (Large mass-density fluctuations have been observed experimentally, see reference 11. Speculations have been put forward that $<(\Delta n)^2>$ may be comparable to N^2.)

For back-scattering, i.e. $\bar{m} = -\bar{k}$, and $\beta = (1/2)\pi$, the scattering cross section reduces to

$$\sigma = \frac{k^4}{16\pi^2} \int_V \int_V < \frac{\Delta K}{K}(\bar{r}_1) \frac{\Delta K}{K}(\bar{r}_2) > e^{i 2\bar{k} \cdot (\bar{r}_1 - \bar{r}_2)} dV_1 \, dV_2 \tag{4}$$

As indicated by Tatarski[4], a change of variables can be introduced as follows

$$\bar{R} = \frac{1}{2}(\bar{r}_1 + \bar{r}_2) ,$$

$$\bar{\rho} = \bar{r}_1 - \bar{r}_2$$

Then

$$\sigma = \frac{k^4}{16\pi^2} \int_V dV_R \int B(\bar{R}, \bar{\rho})\, e^{i 2\bar{k}\cdot\bar{\rho}}\, dV_\rho , \qquad (5)$$

where

$$B(\bar{R}, \bar{\rho}) = < \frac{\Delta K}{K}(\bar{r}_1) \frac{\Delta K}{K}(\bar{r}_2) >$$

Under the conditions that $\omega_p^2 << \omega^2$ and the fluctuation in dielectric constant is due to that of the electron density, as given by Eq. (3), Eq. (5) becomes

$$\sigma = r_e^2 \int_V dV_R \int_V < \Delta n(\bar{R})\, \Delta n(\bar{R} + \bar{\rho}) > e^{i2\bar{k}\cdot\bar{\rho}}\, dV_\rho , \qquad (6)$$

where r_e ($= e^2/c^2 m_e$) is the classical electron radius.

If the scattering volume V is very large, so that the integral in Eq. (5) with respect to ρ can be considered as Fourier integral, the following result is obtained: The contribution to the back-scattering comes from one (and only one) Fourier component of the fluctuations of the dielectric constant, i.e. that component whose wave vector equals to twice the wave vector of the incident electromagnetic wave. This result is generally considered as applicable to scattering problems for which

$$L_n > \frac{\pi}{k} = \frac{\lambda_o}{2} > \ell_n, \qquad (7)$$

where λ_o is the length of the incoming electromagnetic wave, L_n the scale of the largest turbulent "eddies" of the dielectric constant fluctua-

tions, and ℓ_n that of the smallest turbulent "eddies" (these "eddies" are not meant for flow eddies). The size of the scattering volume V must be necessarily larger than L_n^3.

Determination of the correlation function is known to be an outstanding problem in the turbulence theory. A discussion of this problem will be presented in the next section. In the case when the turbulence is homogeneous, i.e. when the correlation function B depends only on $\bar{\rho}$, the expression for σ, Eq. (6), reduces to

$$\sigma = r_e^2 \, V \int_V B(\bar{\rho}) \, e^{i2\bar{k} \cdot \bar{\rho}} \, dV_\rho$$

where $B(\bar{\rho}) = \langle \Delta n(\bar{R}) \, \Delta n(\bar{R} + \bar{\rho}) \rangle$ depends only on $\bar{\rho}$.

If the fluctuations $\Delta n(\bar{R})$ and $\Delta n(\bar{R} + \bar{\rho})$ are not correlated, then $\langle \Delta n(\bar{R}) \, \Delta n(\bar{R} + \bar{\rho}) \rangle$ will be zero, except when $\rho = 0$. In this case, the back-scattering cross section (for a perfect gas, since $\langle (\Delta n)^2 \rangle$ is equal to N/V) becomes

$$\sigma_i = r_e^2 \, N V \, . \tag{8}$$

This is the cross section for "incoherent" back-scattering.[12] It is generally recognized that turbulent scattering is far from being incoherent, since the fluctuations are not quite random. It seems reasonable to suggest the turbulent scattering cross section to be the sum of an incoherent part $a \, r_e^2 \, N V$ and a coherent part $b \, r_e^2 \, N^2 V^2$ where a and b are presumbly independent of the mean electron density N. It will be of interest to see whether this form of turbulent cross section can be justified with our present incomplete understanding of homogeneous turbulence.

III. SOME ASPECTS OF THE WAKE TURBULENCE PHENOMENON

A straightforward way of solving the wake scattering problem is to assume a form for the correlation function in evaluating the integral in Eq. (5). While the correlation coefficient can usually be approxi-

mated by suitable formulas valid under certain circumstances, to predict the intensity of the electron-density fluctuations presents a more difficult problem. In the wake of a hypersonic reentry body, the physical processes by means of which the electron-density fluctuations are produced appear to be sufficiently different from those present in the ionosphere so that an examination of them is warranted.

First, however, the distinction between a turbulent velocity field and a turbulent scalar field should be noted (see reference 10 for the isotropic case). The correlation function for a velocity field is a second-order tensor, while that for a scalar field is a scalar. The physical processes of "dissipation" for these two fields also differ significantly. Therefore, it is important to recognize the difference in form between their correlation or spectrum functions (this point was over looked by several workers). The turbulence scales of the scalar field may also differ from those of the velocity field.

As generally recognized, significant back-scattering by hypersonic wakes apparently takes place only when the wake flow is turbulent. Thus, the fluctuations of electron density are caused by the turbulent motions of the velocity field, i.e., the action of the turbulent eddy motions on the non-uniform distribution of electron density is considered as the mechanism for "turbulent fluctuations" of electron density. By elementary phenomenological consideration, an expression for the fluctuation in terms of the local electron-density gradient, and the scale and intensity of the turbulent motion will be given. It may be mentioned that, in the theory of incoherent scattering, formulas for the electron-density fluctuations have been worked out (see references 12, 13 and 14) by means of more rigorous methods.

In the wake of a high-speed reentry body, the turbulent flow is known to be non-isotropic and non-homogeneous. The turbulent scalar field is also expected to be non-isotropic and non-homogeneous, i.e. the turbulent scales in the longitudinal and transverse directions are different, and the turbulent intensity varies over the wake (decaying on

account of dissipation). It is generally accepted[15], however, that when the turbulent Reynolds numer is high enough, there exists a range of small eddies (high wave-numbers) for which the turbulence is essentially isotropic. If the eddies responsible for the scattering are in this range, the theory of locally isotropic turbulence may be used.

When the length of the incoming electromagnetic wave is comparable to smaller than the wake diameter, however, the larger eddies become the chief sources of scattering. The effect of non-isotropy may become significant, e.g., the scattering cross section may become aspect-angle dependent. Moreover, the condition given by Eq. (7) is no longer satisfied, and the volume of integration Eq. (6) will be a finite one. Consequently, finite-width effect of the wake should appear in the scattering cross section, with the ratio between the wave length and the wake diameter as a parameter. This effect will be treated in the next section.

A prominent feature of turbulent wake flows is the phenomenon of intermittency.[16] It will be shown in the following that electron-density fluctuations, in addition to those caused by the "turbulence", will be introduced by the intermittency.

(1) Effect of Intermittency on Electron Density Fluctuation

According to Townsend[16], the cause of the flow intermittency is the production of a convoluted boundary surface between the turbulent and non-turbulent fluid by the large eddies (of fluid motion). Thus, across the wake an intermittency factor γ is introduced to show statistically the fraction of the time that the flow is found to be turbulent.

Since the hot boundary-layer gas goes into the turbulent core of the wake, electrons produced by thermal ionization will be presented in the turbulent core. In the non-turbulent fluid, the electron density is very low and may be taken as zero.* Intermittency of the turbulent flow

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*Although this condition appears to be valid for slender bodies, in general enthalpy in the outer inviscid wake is not negligible comparable to the inner turbulent wake for hypersonic blunt bodies[17]. The derivation and results to follow can be readily modified to include the electron density distribution in the non-turublent fluid.

will, consequently, introduce at a fixed point fluctuations of electron density, in addition to those present in the entirely turbulent fluid.

Let $\delta(\bar{r}, t)$ denote the probability of turbulence at $\bar{r}$, such that δ is zero in non-turbulent fluid and is unity in turbulent fluid.[16] Time average of δ yields $\gamma(\bar{r})$, the intermittency factor at $\bar{r}$. Let $n(\bar{r}, t)$ be the electron density in the turbulent fluid, $N(\bar{r})$ the mean value of n averaged (in time) over both the turbulent and the non-turbulent fluid, and $\bar{n}(\bar{r})$ the mean value of n in the turbulent fluid. Townsend[16] has shown that $N = \gamma \bar{n}$.

The fluctuation of the electron density with respect to N is equal to $n - N$ in the turbulent fluid, and equal to $-N$ in the non-turbulent fluid. On the average (statistically), the density fluctuation is

$$\Delta n = (n - N)\delta + (-N)(1 - \delta) = n\delta - N. \tag{9}$$

Let $n = \bar{n} + n'$, where n' is the turbulent fluctuation of n (in the conventional sense). Then, the fluctuation of electron density is

$$\Delta n = (\delta - \gamma)\bar{n} + n'\delta . \tag{10}$$

If the fluid is entirely turbulent, i.e., $\delta = \gamma = 1$, Δn becomes n'. When the turbulent flow shows the intermittency phenomenon, according to Eq. (11), additional fluctuation of electron density will occur. Hence, in Eq. (6) the fluctuation in electron density consists of two parts: the turbulent fluctuation and that introduced by the intermittency. The appearance of the mean electron density in the fluctuation term should be noted.

The correlation function for the electron-density fluctuations is

$$<(\Delta n)_1 (\Delta n)_2> = (\gamma_1\gamma_2 - <\delta_1\delta_2>)\bar{n}_1\bar{n}_2 + <\delta_1\delta_2><n_1' n_2'>, \tag{11}$$

(under the assumption that the two functions δ and n' are not correlated)

where the subscripts 1 and 2 refer to points at $\bar{r}_1$ and $\bar{r}_2$, respectively. Let the correlation coefficient χ be defined by

$$\chi = \frac{<\delta_1 \delta_2>}{\gamma_1 \gamma_2} \tag{12}$$

Then, the correlation function can be written as

$$\begin{aligned} <(\Delta n)_1 (\Delta n)_2> &= \gamma_1 \gamma_2 \left\{ (1-\chi) \bar{n}_1 \bar{n}_2 + \chi <n_1' n_2'> \right\} \\ &= (1-\chi) N_1 N_2 + \chi <N_1' N_2'> . \end{aligned} \tag{13}$$

The problem is to find the correlation function for the N' - fluctuations and the correlation coefficient χ.

Fluctuations in dielectric constant may arise from fluctuations in temperature, mass density, and other physical properties of the gas. But for the wake problem, these effects are believed to be relatively unimportant compared to that given in Eq. (11).

(2) Turbulent Fluctuations of Electron Density

The hypersonic wake flows are governed by the processes of mixing between the inner (turbulent) wake and the outer inviscid wake (see reference 17, for example) so that large gradients of the mean velocity, temperature, mass density of the gas, electron density, etc. in the direction transverse to the wake axis exist. Considerable progress has been made recently in treating this problem by phenomenological turbulent-transport theory (references 17, 18, 19 and 20).

The basic physical process for the production of electron-density fluctuations is assumed as follows: Turbulent motion of the flow, in the presence of mean electron-density gradients, will generate irregular fluctuations of electron density. Since the gradient in the longitudinal direction is comparatively very small, the electron-density fluctuation can be expressed as follows

$$N' \sim \ell \frac{\partial N}{\partial r} \quad , \tag{14}$$

where ℓ is a mixing length of the turbulent velocity field. By adopting von Karman's idea,[21] this length is assumed to depend on local turbulent quantities: a turbulence scale (a microscale of turbulence) and a turbulence Reynolds number $u_r \lambda / \nu$ where u_r is the component of velocity fluctuation in the radial direction with respect to the wake axis. In particular, if the relation

$$\ell \sim \frac{\lambda^2 u_r}{\nu} \tag{15}$$

is used, the expression for the electron-density fluctuation becomes

$$N' = c \frac{\lambda^2 u_r}{\nu} \frac{\partial N}{\partial r} , \tag{16}$$

where c is an absolute constant (of order 1 presumbly and to be determined experimentally). The intensity of the fluctuations is

$$\begin{aligned} \overline{N'^2} &= c^2 \frac{\lambda^4 u_r^2}{\nu^2} \left(\frac{\partial N}{\partial r}\right)^2 \\ &= C \frac{\lambda^4 q^2}{\nu^2} \left(\frac{\partial N}{\partial r}\right)^2 \end{aligned} \tag{17}$$

where C is another absolute constant, and $q^2 = \langle (u'^2 + v'^2 + w'^2) \rangle$ is the intensity of the velocity fluctuations.

In considering the density fluctuations in the troposphere and the ionosphere, Villars and Weisskopf[3] took ℓ as a scale of turbulence. Then, the intensity of the electro-density fluctuations would be independent of the intensity of the velocity fluctuations. This result may be valid for the ionosphere, but does not appear to be reasonable for the wake problem. Generally when turbulence decays, the turbulence scale

increases but its intensity of velocity fluctuations decreases. From the theory of decay of isotropic turbulence[15], in the initial stage of decay, the Reynolds number $\lambda q/\nu$ remains constant. Thus, during this stage, $\ell \sim \lambda$. In the final stage of decay, $q \sim t^{-5/4}$, and $\lambda^2 \sim t$ ($t = xU_1^{-1}$, where U_1 is the velocity of the body). Consequently, in the final stage, $\lambda q/\nu \sim t^{-3/4}$. The presence of $\lambda^4 q^2/\nu^2$ in Eq. (17) supports the idea that the intensity of electro-density fluctuations will eventually attenuate as the turbulent energy of the flow decays. In Appendix I, another approach is used in attempting to justify this expression for the intensity of electron-density fluctuations.

In the above consideration, temperature fluctuations are assumed to have no direct bearing on the electron-density fluctuations. In a sense, the turbulence is taken as frozen chemically as far as the electron-density fluctuations are concerned. This assumption is probably reasonable at an altitude around 150,000 ft. in standard atmosphere, at which the recombination rate for ions and electrons is very slow.

Villars and Weisskopf[22] also looked into the possibility that velocity fluctuations will produce pressure fluctuations, which in turn will produce gas-density fluctuations. Assuming that the electron-density fluctuations follow the gas-density fluctuations, they found

$$\overline{N'^2} \approx N^2 \frac{q^2}{v_M^2} \quad , \tag{18}$$

where $u_M^2 = 3p/\rho$. The intensity of fluctuations given by Eq. (18) will be much larger than that given by Eq. (17), if

$$\lambda^2 \left(\frac{\partial \ln N}{\partial r}\right)^2 \left(\frac{v_M \lambda}{\nu}\right)^2 >> 1. \tag{19}$$

This condition is likely valid in hypersonic wakes, since $v_M \lambda/\nu = (q\lambda/\nu)(v_M/q)$ and, for hypersonic wakes, the turbulent Reynolds number is generally much larger than 1, and v_M/q is nearly of order 1.

Compared to an expression for the intensity of electron-density fluctuations of the form $\overline{N'^2} \approx N^2$ (as used by some workers for underdense turbulent scattering), Eq. (17) shows one noteworthy feature, i.e. it contains the square of the turbulent Reynolds number. Therefore, it is a sensitive function of the altitude and the air density. It is proposed that some measurements be carried out to check this point.

(3) The Function $< \delta_1 \, \delta_2 >$

In Eq. (13) for the correlation function of electron-density fluctuations, the function $< \delta_1 \delta_2 >$ appears. This is the "correlation function" for the turbulence probabilities δ_1, and δ_2 at $\bar{r}_1$ and $\bar{r}_2$, respectively. Very little is known at this time about the function δ, but experimentally $< \delta_1 \, \delta_2 >$ should be measurable by making use of Townsend's "δ" signal method[16].

It is simpler to use the correlation function in the form as given by Eq. (13), in which the correlation coefficient $\chi(\bar{R}, \bar{\rho})$ appears. At the present time, however, the only known characteristics about χ appear to be: it approaches 1 when ρ approaches 0 and approaches 0 when ρ is very large. Tentatively, the following form may be used

$$\chi = \exp\left[- \frac{\xi^2 + \eta^2 + \zeta^2}{\alpha^2 R_w} \right], \tag{20}$$

where ξ, η and ζ are the coordinates in $\bar{\rho}$ - space, $R_w(x)$ is the radius (or half-width) of the wake, and α^2 is a constant.

(4) Correlation and Spectrum Functions

In a previous section, the following expression was obtained for the electron-density fluctuation

$$N' = c \, \frac{\lambda^2 u_r}{\nu} \, \frac{\partial N}{\partial r} \tag{16}$$

The question is whether the correlation function for the electron-density fluctuations can now be obtained from $< (u_r)_1 \, (u_r)_2 >$. Since only

phenomenological consideration was employed in deriving Eq. (16) for the excitation mechanism of electron-density fluctuation, this relation applies probably only to the "energy containing eddies" of the N' - spectrum. In the high wave-number portion of the spectrum, dissipation of the turbulent energy and molecular diffusivity control the fine structure of the turbulence but they have not been properly taken into account (see Appendix I). Therefore, Eq. (16) will be used only to find the intensity of the N' - fluctuation, as given in Eq. (17). The correlation function for the N' - fluctuations will be assumed to be

$$< N_1' N_2' > = \overline{N'^2}(\bar{R})\, \chi_N(\bar{R}, \bar{\rho}) \ , \tag{21}$$

where χ_N is the correlation coefficient for the N' - fluctuations. The spectrum function for the N' - fluctuations may be defined by the Fourier integral

$$E_{N,N}(\bar{k}) = \frac{1}{8\pi^3} \int_{-\infty}^{\infty} < N_1' N_2' > e^{-i\bar{k}\cdot\bar{\rho}}\, d\bar{\rho} \tag{22}$$

The inverse of this transform is

$$< N_1' N_2' > = \int_{-\infty}^{\infty} E_{N,N}(\bar{k})\, e^{i\bar{k}\cdot\bar{\rho}}\, d\bar{k}$$

When the scale of the electron-density fluctuations responsible for the scattering ($\sim \lambda_o/2$, where λ_o is the incident electromagnetic wave length) is in the high wave-number range, results from turbulence theory of local isotropy may be used for the correlation and spectrum functions. It is necessary to take $\gamma = 1$, so that there is no intermittency and $n' = N'$. If, in addition, there exists an inertial subrange in the N' - spectrum, the problem becomes more tractable and has been treated by Batchelor[23]. The relative magnitude of the molecular diffusivity D and the kinematic viscosity ν was found to be the decisive parameter in determining the

form of the n' - spectrum. If the three-dimensional velocity spectrum function is of the form as follows (Kolmogoroff form)

$$E(k, t) \sim \epsilon^{2/3} k^{-5/3} \tag{23}$$

then, if $\nu \sim D$, in the same k-range the corresponding three-dimensional spectrum function E_n, defined by $E_n = 4\pi k^2 E_{n,n}$, will have the form

$$E_n(k, t) \sim \epsilon_n \epsilon^{-1/3} k^{-5/3} \tag{24}$$

In the above expressions, ϵ is the dissipation of the turbulent motion by viscosity, while ϵ_n is the "dissipation" of the n' - fluctuations by the molecular diffusivity. Both these two functions may be expressed in terms of their respective intensity and scale of turbulent fluctuations. The scales of turbulence for the velocity and n' - fluctuations are also of the same order (see reference 10). Under this circumstance, the condition required for the presence of an inertial subrange in the n' - spectrum is

$$\frac{\lambda q}{\nu} >> 1 \tag{25}$$

Since the size Λ of the "energy containing" eddies can be assumed to be of the order of wake radius, by using the relation between the turbulent Reynolds numbers (reference 10)

$$\frac{\Lambda q}{\nu} = \frac{A}{15}\left(\frac{\lambda q}{\nu}\right)^2 \tag{26}$$

an estimate for the value of $\lambda q/\nu$ in the turbulent wake can be made. The precise value for $\lambda q/\nu$ at which an inertial subrange will be in existence is not known, but a value of 10^3 is the likely minimum. (The value of the constant A in the above equation is about 1).

When the wave length λ_o is comparable to the wake width, the larger eddies assume the role of scattering, and some non-isotropic effects of turbulence may be expected. To evaluate these effects, the

correlation function will be assumed to be of a form which allows the turbulence scales in the longitudinal (along the wake axis) and transverse directions to be different. This idea is adopted from Booker's work[24] on radar reflections from the aurora. Accordingly, the correlation function for the N' - fluctuations is assumed to be of the Gaussian form

$$< N_1' \, N_2' > = \overline{N'^2} \, (\bar{R}) \exp \left[- \frac{1}{2} \left(\frac{\xi^2}{L^2} + \frac{\eta^2 + \zeta^2}{T^2} \right) \right] , \qquad (27)$$

where ξ, η and ζ are the coordinates in $\bar{\rho}$ - space, L and T are the correlation lengths in the longitudinal and transverse directions, respectively, and $\overline{N'^2}$ is the intensity of N' - fluctuations at $\bar{R} = (1/2)\,(\bar{r}_1 + \bar{r}_2)$, as given by Eq. (17). The Gaussian correlation function is believed to be valid at very low turbulence Reynolds numbers (corresponding to the final period of decay), but for large Reynolds numbers, an exponential function may be a better approximation (see p. 173 of reference 10). There is some uncertainty as to the values of L and T for the wake, both of which may depend on $\bar{R}$.

As pointed out by Tatarski,[25] use of spectrum functions for the turbulent scattering problem offers some advantage, when the precise form of the spectrum function is known, (in the inertial subrange for example). The important question of how the scattering cross section depends on the frequency of the incident electromagnetic wave can be answered without any ambiguity. For turbulence at smaller turbulent Reynolds numbers (so that no subinertial range exists), and with non-isotropic effects, the exact form of the correlation or spectrum function is not known. Use of a Gaussian correlation function as given in Eq. (27) cannot be expected to predict, in general, the exact frequency-dependence of the scattering cross section.

(5) Some Characteristics of the Turbulent Wake

In turbulent wake flows, the concept of self-preservation as introduced by Townsend[16, 17] has been found to be particularly useful. For an incompressible axisymmetric wake flow, the mean velocity profile

and the distribution of the turbulent intensity can be assumed to be of the following form

$$U = U_1 - u_o f(r/\ell_o) , \tag{28}$$

$$\overline{u^2} + \overline{v^2} + \overline{w^2} = u_o^2 g(r/\ell_o) , \tag{29}$$

where U_1 is the free-stream velocity, $r = (y^2 + x^2)^{1/2}$, and u_o and ℓ_o, suitably chosen scales of velocity and length, are functions of t only. u_o is called the defect velocity, when the function $f(r/\ell_o)$ is chosen to be unity at $r = 0$, (ℓ_o can be identified as the wake width). It is found that self-preserving solutions are admissible by the equations of motion, and

$$\frac{u_o}{U_1} \sim \left(\frac{d}{x - x_o}\right)^{2/3} \tag{30}$$

$$\frac{\ell_o}{d} \sim \left(\frac{x - x_o}{d}\right)^{1/3} \tag{31}$$

where d is a typical length of the body and x_o is a virtual origin of the flow.

Wakes generated by hypersonic reentry bodies are known to differ in some respects from their low-speed counterparts[17]. Consideration must be given to a turbulent inner wake (when transition has taken place) and an inviscid outer wake. Not only compressibility effects but chemical reactions involving dissocation, ionization, recombination, etc. must be taken into account in determining the mean profiles of velocity, temperature, electron density, etc. Progress has been made on this complicated problem (see references 17, 19, 20, for example), but many uncertainties remain to be resolved. In addition, very little knowledge is available on the turbulence structure in hypersonic wake flows. Additional theoretical and experimental studies are needed.

As far as the scattering phenomenon is concerned, the most important physical quantity of the wake is the electron-density (mean)

distribution. According to Eq. (17), the intensity of the N' - fluctuations depends on the gradient of this distribution. A "wake length" defined as the distance from the body to the position on the wake-axis at which the plasma frequency is equal to the incident electromagnetic-wave frequency is found generally to be a useful reference length (references 17 and 26). At the present time, uncertainties in the electron-density distribution and the wake length are mainly due to non-equilibrium chemical reactions, conditions at the wake neck region, turbulent transport properties, and effect of impurities present in the air. It is important to note, however, that by referring to a band frequency of the radar, an overdense core surrounded by an underdense outer portion is a significant feature of the wake. A scattering model with this feature is not available yet.

There are strong experimental evidences that the width of the turbulent inner wake can be approximated by the power relation $(x - x_o)^{1/3}$ as in the incompressible case (see references 17, 27 and 28). If the turbulence scales (microscales) are taken as constant across the width of the wake, and, in addition, assumed to be proportional to the wake width, then both L and T in Eq. (27) may be considered as proportional to $(x - x_o)^{1/3}$. This would be in variance with that for an isotropic decaying turbulence, for which $\lambda \sim t^{1/2} \sim (x - x_o)^{1/2}$. However, there is some experimental evidence that for axially symmetric wakes, the "microscale of turbulences" does not vary according to $x^{1/2}$ (see reference 29).

For incompressible wake flows (two-dimensional), Townsend[16] has found experimentally that the turbulent intensity of the velocity fluctuations in the turbulent fluid is almost uniform across the wake, with contributions mainly from the smaller eddies. This result should also hold true for hypersonic wakes, in which the turbulence mechanism is expected to be similar to the low-speed case in the coordinate system fixed to the ground. Hence, according to Eq. (17), (in which λ is replaced by T to allow for the non-isotropic effect in turbulence scales), the variation of the intensity of the N' - fluctuations follows that of

$(\partial N/\partial r)^2$ in both the longitudinal and the transverse directions in the wake. This result points to the importance of having an accurate determining of the mean electron-density distribution in hypersonic wake.

(6) Summary

The correlation function for the electron-density fluctuations has been found to be of the form

$$<(\Delta n)_1 (\Delta n)_2> = (1 - \chi) N_1 N_2 + \overline{N'^2} \chi \chi_N \tag{32}$$

where N denotes the mean electron density, $\overline{N'^2}$ is the intensity of the electron-density fluctuations as given by Eq. (17), χ is the correlation coefficient for the flow intermittency, and χ_N that for the electron-density fluctuations. Subscripts 1 and 2 refer to points at $\bar{r}_1$, and $\bar{r}_2$, for which the double correlation coefficients are defined. Suggested forms for the coefficients χ and χ_N are Eq. (20) for χ, and Eq. (24) (for the spectrum function) or Eq. (27) for χ_N. It may be added that the expressions for the electron fluctuation and the correlation function apply both to underdense and overdense plasmas.

IV. SCATTERING CROSS SECTION

The correlation function for the electron-density fluctuations given by Eq. (32) may be submitted into Eq. (6) to obtain an expression for the scattering cross section in a form more susceptible to physical interpretation. It is convenient to separate the cross section into two parts:

$$\sigma = r_e^2 (\sigma_a + \sigma_b) , \tag{33}$$

where

$$\sigma_a = \int_V dV_R \int_V \left\{ 1 - \chi(\bar{R}, \bar{\rho}) \right\} N(\bar{r}_1) N(\bar{r}_2) e^{i2\bar{k} \cdot \bar{\rho}} dV_\rho , \tag{34}$$

$$\sigma_b = \int_V \overline{N'^2} (\bar{R}) dV_R \int_V \chi(\bar{R}, \bar{\rho}) \chi_N (\bar{R}, \bar{\rho}) e^{i2\bar{k} \cdot \bar{\rho}} dV_\rho , \tag{35}$$

and $2\bar{R} = \bar{r}_1 + \bar{r}_2$, $\bar{\rho} = \bar{r}_1 - \bar{r}_2$. If the effect of the flow intermittency is omitted, χ becomes unity, and σ_a vanishes. Then σ_b reduces to

$$\sigma^* = \int_V \overline{n^2}(\bar{R})\, dV_R \int_V \chi_n(\bar{R}, \bar{\rho})\, e^{i2\bar{k}\cdot\bar{\rho}}\, dV_\rho \quad . \tag{36}$$

In considering the characteristics of the scattering cross section, of particular interest are its dependence on the wave number (or frequency) of the incident electromagnetic wave, its dependence on the aspect angle θ (Figure 1), and how the various physical properties of the wake affect the cross section. Evidently, it is necessary to know the mean electron-density distribution $N(\bar{r})$, the distribution of intensity of the velocity fluctuations q^2, the turbulence scales L and T, and the correlation coefficients χ and χ_N. At the present time, none of these quantities can be predicted with certainty. Therefore, the results obtained in the following are intended to show only qualitatively the essential features of the phenomenon of scattering by turbulent wakes.

(1) <u>Dependence on the Wave Number and the Aspect Angle</u>

By referring to Figure 1, write

$$\bar{k} = k(\bar{i} \cos\theta + \bar{j} \sin\theta) \tag{37}$$

where $\bar{i}$ and $\bar{j}$ are unit vectors in x and y (or ξ and η) directions, respectively. If the volume of integration in Eqs. (34) and (35) is very large, the dependence of the cross section on k and θ can be found by using Fourier integrals. Since the functions N, χ and χ_N attenuate at large ρ, from the theory of Fourier integrals (see Titchmarch,[30] for example) the scattering cross section must necessarily decrease as the wave number k (at least, for sufficiently large values of R) of the incident electromagnetic wave is increased. In addition, if the attenuation rate is not isotropic in the ρ - space, the cross section will become aspect-angle dependent.

The difference between σ_a and σ_b in their dependence on k and θ is to be noted. Consider first the following integral in σ_a:

$$\int_V dV_R \int N(\bar{r}_1) N(\bar{r}_2) e^{i2\bar{k}\cdot\bar{\rho}} dV = \left| \int_V N(x,y,z) e^{i2k(x\cos\theta + y\sin\theta)} dxdydz \right| \quad (38)$$

Since N is expected to attenuate strongly in y, but weakly in x, the above integral will atenuate strongly in k sin θ, and weakly in k cos θ. Thus, σ_a will depend very strongly on k for $\theta \sim (1/2)\pi$, but will be less sensitive to a change in k at small aspect angles. On the other hand, in σ_b the main contribution to its k and θ dependence is due to the difference in turbulence scales between the transverse and axial directions. Since this difference in scales is minor compared to that for the attenuation rates of N in x and y directions, the aspect-angle dependence of σ_b will be less prominent. The simplest case is obtained, when the turbulence is isotropic and the three-dimensional spectrum function is of the form given by Eq. (24), i.e., $E_n \sim k^{-5/3}$, and the flow intermittency is omitted. Then, from Eqs. (22) and (36) and the relation $E_n = 4\pi k^2 E_{n,n}$,

$$\sigma \sim k^{-11/3} \quad (39)$$

There will be no dependence of σ on the aspect angle, if the turbulence is isotropic the scattering volume is infinitely large.

Based on the expressions (33), (34) and (35), the dependence of the scattering cross section on k and θ is, in general, too complicated to be expressed in a simple manner. Further analysis, both analytically and numerically is needed.

(2) Some General Characteristics

One of the main points introduced in the present study is the intermittency phenomenon of turbulent flows. Since the correlation coefficient χ for the intermittency is always smaller than 1, the effect of the flow intermittency is:

(a) to introduce a contribution σ_a, which depends directly on the mean electron-density distribution N,

(b) to reduce the contribution from the N′ - fluctuations, as seen by comparing σ_b to σ^*.

Because of the lack of knowledge about χ and the mathematical difficulty in evaluating the integrals in Eq. (34), it is not possible to determine, in general, the comparative magnitude between σ^* and $\sigma_a + \sigma_b$, and also that between σ_a and σ_b. A simple analysis by using mean electron-density distributions of an exponential type is given in Appendix II.

The expression (38), multiplied by r_e^2, would appear to be a "coherent" scattering cross section.[31] However, the contribution σ_a arises solely from the flow intermittency, and vanishes if χ is taken to be unity. In addition, N is the mean electron density averaged over both the turbulent and non-turbulent fluids, i.e., the intermittency phenomenon is also included in N.

For the wake problem, a more explicit form of the scattering cross section $\sigma = r_e^2 \sigma^*$ without intermittency ($\gamma = 1$, $n = N$ and $n' = N'$) can be obtained by using Eq. (17) for $\overline{n'^2}$ and Eq. (27) for the correlation coefficient χ. Then, from Eq. (36), if the volume of integration is extended to infinity,

$$\sigma^* = \int_V \overline{n'^2}\, dV_R \int_V \exp\left\{-\frac{1}{2}\left(\frac{\xi^2}{L^2} + \frac{\eta^2 + \zeta^2}{T^2}\right) + i2k(\xi \cos\theta + \eta \sin\theta)\right\} d\xi\, d\eta\, d\zeta$$

$$= C_1 (2\pi)^{3/2} \int_V \left(\frac{\partial \bar{n}}{\partial r}\right)^2 L\, T^2 \exp\left\{-\frac{8\pi^2}{\lambda_o^2}\left(L^2 \cos^2\theta + T^2 \sin^2\theta\right)\right\} dV_R \,, \tag{40}$$

where $C_1 = C\, T^4 q^2/\nu^2$ is a constant (see p. 10). Let

$$\bar{n}(x, r) = n_o F(x)\, G(s) \tag{41}$$

where $n_o F(x)$ is the electron-density distribution along the wake axis, $s = [r/d^{2/3} (x - x_o)^{\frac{1}{3}}]$, both F and G are non-dimensional, and $G(o) = 1$. The final form of σ^* may be written as follows:

$$\sigma^* = C_1 L_w L_o T_o^2 n_o^2 I_1 I_2 , \tag{42}$$

where L_w is the wake length defined in a proper way, L_o and T_o are the typical scales of turbulence in the axial and transverse directions, respectively, and

$$I_1 = 2\pi \int_o^\infty G'^2 (s)\, s\, d\, s$$

$$I_2 = \int_{x_o/L_w}^{1} F(x) \frac{L\, T^2}{L_o\, T_o^2} \exp\left\{- \frac{8\pi^2}{\lambda_o^2} (L^2 \cos^2 \theta + T^2 \sin^2 \theta)\right\} dx$$

I_1 is a pure constant, while I_2 is non-dimensional but depends on θ, L_o/λ_o, T_o/λ_o, and L_w/x_o.

Since $C_1 = C\, T_o^2 (T^4 q^2/\nu^2)$, the cross section σ^* given by Eq. (41) is proportional to T_o^4, and is also sensitive to the turbulence Reynolds number $T\, q/\nu$. The dependence on the aspect angle θ vanishes when the turbulence scales L and T are the same.

To be able to evaluate the results obtained above, it is necessary to determine the quantities: L_w - the wave length, $n_o F(x)$ - the electron density distribution along the wake axis, L_o and T_o - the turbulence scales, $T\, q/\nu$ - the turbulence Reynolds number, I_1 and I_2 , and the proportional constant C. How these quantities vary with altitude and with the velocity and geometry of the body will be of particular interest. At the present time, there exists considerable uncertainty in the electron-density distributions in the wake. Results for chemically reacting wakes obtained by Bloom and Steiger[19] show that the electron density is not sensitive to

changes in altitude. Lees and Hromas[17] gave the results, based on frozen and equilibrium conditions, that a change in altitude from 200,000 ft. to 100,000 ft. would yield electron density higher by about two orders of magnitude. Evidently, much work in this area is needed, before meaningful correlation with experimental measurements can be carried out.

(3) Effect of Finite Wake Width

Now the influence of finite width of the wake on the aspect-angle dependence will be treated in an approximate manner, with intermittency omitted. Again consider

$$\sigma^* = \int_V \overline{n'^2}\, dV_R \int_V \exp\left\{-\frac{1}{2}\left(\frac{\xi^2}{L^2} + \frac{\eta^2+\zeta^2}{T^2}\right) + i2\bar{k}\cdot\bar{\rho}\right\} dV_\rho \ .$$

Assume the component of the correlation distance in the transverse direction is limited by the width of the wake. Then the integral with respect to ρ becomes

$$(2\pi)^{\frac{1}{2}}\, L \exp\left\{-8\pi^2 (L^2/\lambda_o^2)\cos^2\theta\right\} \int\int_{-R_w}^{R_w} \exp\left\{-\frac{\eta^2+\zeta^2}{2T^2} + i\,\frac{4\pi\eta}{\lambda_o}\sin\theta\right\} d\eta\, d\zeta\ ,$$

where R_w is the radius of the wake. This integral can be evaluated approximately to yield

$$4(2)^{\frac{1}{2}}\,\pi\, L\, T^2 \exp\left\{-8\pi^2(L^2/\lambda_o^2)\cos^2\theta\right\} \mathrm{erf}(R_w/2^{\frac{1}{2}}T)\cdot I$$

where

$$I = \mathrm{erf}\left(\frac{R_w}{2^{\frac{1}{2}}T}\right) - 32\pi^2(\sin^2\theta)\left(\frac{T}{\lambda_o}\right)^2 \left\{\mathrm{erf}\left(\frac{R_w}{2^{\frac{1}{2}}T}\right) - \frac{R_w}{2^{\frac{1}{2}}T}\, e^{-\frac{R_w^2}{2T^2}}\right\} + \ldots$$

When θ is small, higher order terms in $\sin^4\theta$, etc. can be neglected in I. The above result indicates that the back-scattering cross section will be

reduced when the wake width is finite. In addition, since I decreases when θ is increased, the effect of finite wake width introduces aspect-angle dependence into the scattering cross section. The fact that reduced cross section for finite wake width occurs with increasing aspect angle seems to be reasonable on an intuitive argument, namely, the incident electromagnetic wave will be able to interact with more eddies at small incidence angles than at larger ones. This is another aspect of the turbulent scattering. In the latter case, the scattering cross section is proportional linearly to the number of electrons and independent of the aspect angle and the wave frequency. In the above consideration, the beam width of the incident wave is assumed to be infinite.

When $R_w \to \infty$, the integral becomes

$$(2\pi)^{3/2} L T^2 \exp\left\{- \frac{8\pi^2}{\lambda_o^2} (L^2 \cos^2 \theta + T^2 \sin^2 \theta)\right\},$$

which (the same as given in Eq. (40)) has been given by Booker in reference 24. This result written in the form

$$(2\pi)^{3/2} L T^2 \exp\left\{- \frac{8\pi^2}{\lambda_o^2} (L^2 - T^2) \cos^2 \theta - 8\pi^2 \left(\frac{T}{\lambda_o}\right)^2\right\}$$

shows the aspect-angle dependence of non-isotropic scattering by a large volume. If $L > T$, the cross section increases with the aspect angle in contrast to the case of a wake with finite width. Therefore, the effect of finite width is found to be quite significant.

V. CONCLUSIONS AND RECOMMENDATION FOR FURTHER WORK

The main results developed in this study are:

(1) The phenomenon of flow intermittency, as generally recognized to be a prominent feature of turbulent wake flows, will introduce electron-density fluctuations, in addition to those caused by turbulent fluctuations.

(2) The electron-density fluctuations caused by turbulent motions are shown to depend on the gradient of the mean electron-density distribution, a turbulent Reynolds number, and a turbulence scale,

(3) By using the above two results, an expression for the back-scattering cross section for under dense wakes is given. Two contributions to this cross section are $r_e^2 \sigma_a$ and $r_e^2 \sigma_b$, given by Eqs. (34) and (35) . A significant point is that σ_b depends on the turbulent Reynolds number but σ_a does not. Thus, at higher altitudes σ_a may be expected to be domirant, while at lower altitudes σ_b will become more important.

The phenomenon of intermittency is found to introduce significant aspect-angle dependence into the scattering cross section. Other effects discussed in some detail are those due to finite wake width and the difference in turbulence scales in the axial and transverse directions in the wake.

Mathematical difficulties and lack of knowledge about the phenomenon of the flow intermittency make a thorough analysis of these effects not feasible at this time. It is recommended that

(1) further theoretical and numerical anslyses of the scattering problem initiated in this study be continued in order to develop the results obtained so far into more useable form, and to correlate them with experimental work,

(2) to study, probably by experimental means, the phenomenon of flow intermittency, e.g., to measure more accurately the intermittency factor for hypersonic wakes, its dependence on the angle of attack of the hypersonic body, and the correlation coefficient for the flow intermittency,

(3) to undertake basic studies of the turbulence problem, e.g., the turbulence theory, turbulent transport properties, intensity of turbulent velocity fluctuations, scales of turbulence, and other turbulent statistical properties of hypersonic wakes, and

(4) to carry out an analysis of turbulent scattering for hypersonic wakes based on the model that an overdense core is surrounded by an underdense outer wake.

APPENDIX I

In Section III. 2, the following expression for the intensity of the turbulent electron-density fluctuations is given

$$\overline{N'^2} = C \frac{\lambda^4 q^2}{\nu^2} \left(\frac{\partial N}{\partial r}\right)^2 \tag{17}$$

Now, a partial justification is attempted here. For the consideration of the excitation of the electron-density fluctuation by the turbulent velocity field, effect of the gradient of the mean flow velocity is considered as small compared to that of the mean electron-density distribution. Assume the electron density distribution is controlled by the diffusion process, and use a coordinate system which follows the local mean motion of the fluid particle. The equation governing the fluctuation of electron density is

$$\frac{\partial N'}{\partial t} + u_i \frac{\partial N}{\partial x_i} = D \frac{\partial^2 N'}{\partial x_i \partial x_i} + \left[u_i \frac{\partial N'}{\partial x_i} - \langle U_i \frac{\partial N'}{\partial x_i} \rangle \right], \tag{A-1}$$

where $N'(x_i, t)$ is the fluctuation of the electron density, $N(x_i)$ is the mean electron density, u_i is the velocity fluctuation and D is the molecular diffusion coefficient. The second term at the right-hand side of the above equation will be considered as small and omitted. Similar consideration has been developed in reference 32.

Consider an initial value problem: Let $N' = 0$, at $t = 0$. Given u_i and N, determine N' as governed by the differential equation (A-1). Solution of N' at large t will be the desired electron-density fluctuation, since the excitation term $u_i \, \partial N/\partial x_i$ and the dissipation by diffusion will balance each other. However, since little is known about $u_i(x_i, t)$ and the analytical behavior of N', the equation (A-1) is replaced by

$$\frac{\partial N'}{\partial t} = S - \frac{D}{\lambda^2} N' , \tag{A-2}$$

where S represents the excitation term and λ is a turbulence scale, both assumed to be time-independent for the present analysis. The solution of Eq. (A-2) is

$$N' = \frac{S\lambda^2}{D}\left[1 - \exp\left(-\lambda^2 t/D\right)\right] \tag{A-3}$$

Thus, at large t,

$$N' \sim -\frac{\lambda^2 u_i}{D}\frac{\partial N}{\partial x_i} = -\frac{\lambda^2 u_i}{\nu}\frac{\partial N}{\partial x_i}\frac{\nu}{D}\ , \tag{A-4}$$

which is of the same form as Eq. (16). The constant C in Eq. (16) is thus probably of the order ν/D, which is independent of the gas density. The intensity of the fluctuations will be essentially the same as given by Eq. (17) for the wake problem.

If Eq. (A-1) is multiplied by N' and the average is taken, the following equation is obtained (only significant terms are kept)

$$\frac{1}{2}\frac{\partial \overline{N'^2}}{\partial t} + \overline{u_i N'}\frac{\partial N}{\partial x_i} = D\overline{\left(\frac{\partial N'}{\partial x_i}\frac{\partial N'}{\partial x_i}\right)} \tag{A-5}$$

Now introduce the eddy diffusivity D_T by the relation

$$-\overline{u_i N'} = D_T\frac{\partial N}{\partial x_i} \tag{A-6}$$

Eq. (A-5) becomes

$$\frac{1}{2}\frac{\partial \overline{N'^2}}{\partial t} = D_T\left(\frac{\partial N}{\partial x_i}\frac{\partial N}{\partial x_i}\right) - D\overline{\left(\frac{\partial N'}{\partial x_i}\frac{\partial N'}{\partial x_i}\right)} \tag{A-7}$$

Assume Eq. (A-7) can be treated in a manner similar to Eq. (A-1). The result is

$$\overline{N'^2} = \frac{D_T}{D}\lambda^2\left[1 - \exp\left(\sim\lambda^2 t/D\right)\right]\frac{\partial N}{\partial x_i}\frac{\partial N}{\partial x_i} \tag{A-8}$$

Thus, the intensity of electron-density fluctuations is, approximately,

$$\overline{N'^2} = \frac{D_T \lambda^2}{\nu} \frac{\partial N}{\partial x_i} \frac{\partial N}{\partial x_i} \frac{\nu}{D} . \qquad \text{(A-9)}$$

Comparison of Eq. (A-9) with Eq. (17) shows that the eddy diffusivity is of the form

$$D_T \approx \frac{q^2 \lambda^2}{\nu} \frac{\nu}{D} , \qquad \text{(A-10)}$$

which is analogous to von Karman's concept of eddy viscosity.[21]

Following Prandtl's idea, for wake flows the eddy viscosity coefficient is generally assumed to be given by[17]

$$\nu_T \sim (\Delta U) \cdot \ell_o , \qquad \text{(A-11)}$$

where ΔU is the greatest difference in the mean velocity and ℓ_o is the width of the wake. Compared to the thermodynamic transport coefficients D and ν, the difference in form between D_T and ν_T, as given by (A-10) and (A-11), respectively, is remarkable. However, the only justification usually given for accepting (A-11) is that satisfactory mean velocity distributions in the mixing zone can be obtained by using ν_T of this form. It remains to be seen, of course, whether D_T in the form (A-10) will lead to satisfactory results for the electron-density fluctuations. A point of interest is that D_T given by (A-10) is a sensitive function of altitude, since ν^{-1} depends linearly on the ambient gas density while ν/D is nearly a constant.

APPENDIX II

It would be useful to have some idea about the relative magnitude of the two contributions, σ_a and σ_b, in the scattering cross section as given by Eq. (33). Lack of knowledge about the flow intermittency and mathematical difficulties prevent an evaluation in a general manner. In this appendix, a numerical example is given for mean electron-density distribution of the exponential type

$$N(x_i) = N_o \exp\left[-\left(\frac{x^2}{a^2} + \frac{y^2+z^2}{b^2}\right)\right] \tag{A-12}$$

The correlation coefficients for the N' - fluctuations and the flow intermittency are assumed to be

$$\chi_N = \exp\left[-\left(\frac{\xi^2}{L^2} + \frac{\eta^2+\zeta^2}{T^2}\right)\right], \tag{A-13}$$

$$\chi = \exp\left[-\left(\frac{\xi^2}{P^2} + \frac{\eta^2+\zeta^2}{Q^2}\right)\right] . \tag{A-14}$$

Substitution and evaluation of the integrals yield

$$\sigma_a = \frac{3}{8}\pi^3 N_o^2 a^2 b^4 \exp\left[-2k^2(a^2\cos^2\theta + b^2\sin^2\theta)\right] \times$$
$$\left\{1 - \frac{PQ^2}{(P^2+2a^2)^{\frac{1}{2}}(Q^2+2b^2)} \exp\left[2k^2\left(\frac{2a^4\cos^2\theta}{P^2+2a^2} + \frac{2b^4\sin^2\theta}{Q^2+2b^2}\right)\right]\right\}, \tag{A-15}$$

$$\sigma_b = \frac{1}{2^{5/2}}\pi^3 N_o^2 C\left(\frac{Tq}{\nu}\right)^2 \frac{a\,L\,T^4\,PQ^2}{(L^2+P^2)^{\frac{1}{2}}(T^2+Q^2)} \exp\left[-k^2\left(\frac{L^2P^2\cos^2\theta}{L^2+P^2} + \right.\right.$$
$$\left.\left.\frac{T^2Q^2\sin^2\theta}{T^2+Q^2}\right)\right]. \tag{A-16}$$

The ratio between these two contributions is

$$\frac{\sigma_b}{\sigma_a} = C\left(\frac{Tq}{\nu}\right)^2 \frac{LT^4}{3a\,b^4} \frac{2^{1/2} PQ^2}{(L^2+P^2)^{\frac{1}{2}}(T^2+Q^2)} \exp\left\{-k^2\left[\left(\frac{L^2P^2}{L^2+P^2}-2a^2\right)\cos^2\theta+\left(\frac{T^2Q^2}{T^2+Q^2}-2b^2\right)\sin^2\theta\right]\right\}$$

$$\times\left\{1-\frac{PQ^2}{(2a^2+P^2)^{\frac{1}{2}}(2b^2+Q^2)}\exp\left[2k^2\left(\frac{2a^4\cos^2\theta}{2a^2+P^2}+\frac{2b^4\sin^2\theta}{2b^2+Q^2}\right)\right]\right\}^{-1} \qquad \text{(A-17)}$$

If all the scales L, T, a, b, P and Q are of the same order of magnitude, σ_b is probably larger than σ_a because C is of order 1 and $(Tq/\nu) > 1$. If P and Q are very small, then σ_b/σ_a will be small.

The more significant difference between σ_a and σ_b is that σ_b depends on the turbulent Reynolds number Tq/ν, while σ_a does not. Thus, at high altitudes σ_a may be the dominant contribution, and at relative lower altitudes σ_b becomes the dominant one.

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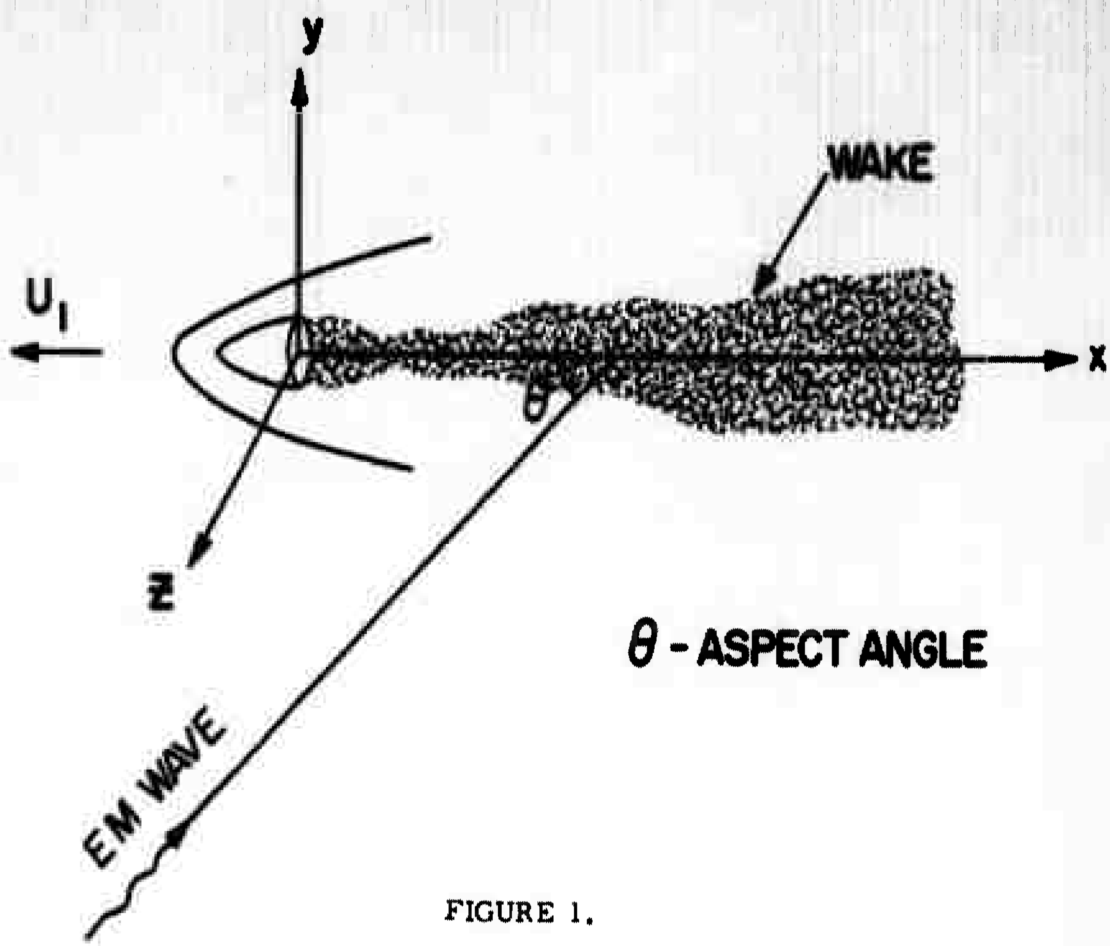

FIGURE 1.

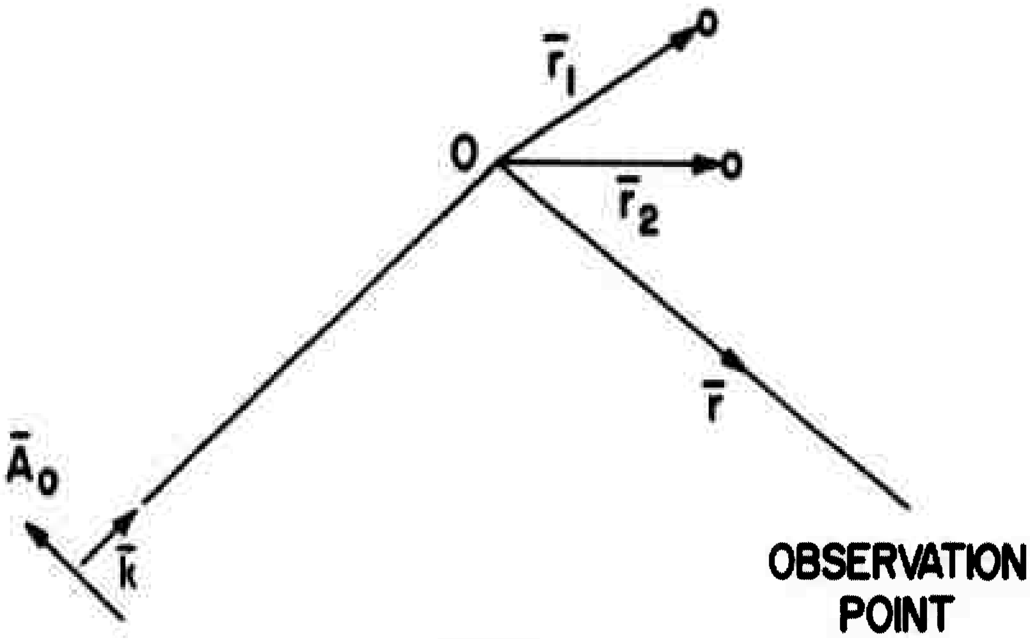

FIGURE 2.

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		DATE Dec. 1963
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SUMMARY

The present analytical study of the turbulent scattering of electromagnetic waves is directed to certain features of the phenomenon considered as peculiar to hypersonic wakes. Effects of flow intermittency, non-isotropy of the turbulence structure, and finite width of the wake on the scattering cross section in its frequency and aspect-angle dependence are of primary concern.

Phenomena of turbulence revelant to the scattering problem are briefly discussed. In particular, it is shown that electron-density fluctuations of a turbulent nature, in addition to those caused by "turbulence", will be produced by the intermittency behavior of turbulent wake flows. This fluctuation is found to depend on the mean electron-density distribution, and Townsend's intermittency "δ" function.

Application of the above considerations to turbulent scattering by underdense wakes has been made. In contrast to the ionospheric scattering, two contributions to the scattering cross section are obtained in the present case: the first one arises from the intermittency phenomenon and vanishes if the intermittency is omitted, and the second one is due to "turbulent" fluctuations. The frequency-dependence of these two contributions are seen to be quite different. It is further shown that flow intermittency, non-isotropy of turbulence structure, and finite width of the wake will introduce aspect-angle dependence into the scattering cross section. A possible form of the total scattering cross section for hypersonic wakes is also given.

KEY WORDS

Scattering cross-section, electron density

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